Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS	**9709/12**
Paper 1 Pure Mathematics 1	**October/November 2022**
MARK SCHEME	
Maximum Mark: 75	

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2022 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	Mid-point AB is $\left(\frac{10+5}{2},\frac{2-1}{2}\right)\left[=\left(\frac{15}{2},\frac{1}{2}\right)\right]$	**B1**	Accept unsimplified.
	Gradient of AB = $\frac{-1-2}{10-5}=\frac{-3}{5}$ Gradient perpendicular = $\frac{5}{3}$	**M1**	For use of $\frac{\text{Change in y}}{\text{Change in } x}$, condone inconsistent order of x and y, and $m_1m_2 = -1$.
	$\frac{\boldsymbol{y}-\frac{1}{2}}{\boldsymbol{x}-\frac{15}{2}}=\frac{5}{3}$ $\left[\boldsymbol{y}-\frac{1}{2}=\frac{5}{3}\left(\boldsymbol{x}-\frac{15}{2}\right)\right]$	**A1**	OE ISW Any correct version e.g. $y=\frac{5}{3}x-12$ or $5x-3y=36$.
		3	
1(b)	[Radius =] $\sqrt{34}$ or 5.8 AWRT or [(radius)^2 =] 34	**B1**	Sight of $\sqrt{34}$ or 34. Condone confusion of r and r^2.
	$(x-5)^2+(y-2)^2$	**B1**	Sight of $(x-5)^2+(y-2)^2$
	$(\boldsymbol{x}-5)^2+(\boldsymbol{y}-2)^2=34$	**B1**	CAO ISW
	Alternative method for Question 1(b)		
	$x^2+y^2-10x-4y$	**B1**	
	$[c=]5$ or $[c=]-5$	**B1**	Substitution of (10, –1) into $x^2+y^2-10x-4y+c=0$.
	$\boldsymbol{x}^2+\boldsymbol{y}^2-10\boldsymbol{x}-4\boldsymbol{y}-5=0$	**B1**	
		3	

Question	Answer	Marks	Guidance
2	$2a - a = a^2 - 2a$	**B1**	OE An unsimplified correct equation in a or d only, e.g. $a^2 + a = 4a$. Can be implied by correct values for a or d.
	$a = 3$ or $d = 3$	**B1**	Condone 'extra' solution of $a = 0$ or $d = 0$.
	$a = 3$ and $d = 3$	**B1**	SOI
	$S_{50} = \frac{50}{2}\left(2 \times their\, a + 49 \times their\, d\right)$	**M1**	May be done using 50th term (=150). Their a and d must be numerical.
	3825	**A1**	ISW **SC B2** for $1275a$ or $1275d$
		5	

Question	Answer	Marks	Guidance
3(a)	$k^2 - 4 \times 8 \times 2$ [< 0]	**M1**	Use of $b^2 - 4ac$ but not just in the quadratic formula.
	$-8 < k < 8$ or $-8 < k$, $k < 8$ or $\lvert \boldsymbol{k} \rvert < 8$ or $(-8, 8)$	**A1**	Condone '$-8 < k$ or $k < 8$', '$-8 < k$ and $k < 8$' but not $\sqrt{64}$.
		2	

Question	Answer	Marks	Guidance
3(b)	$2(4\cos\theta-1)(\cos\theta-1)$ or $(4\cos\theta-1)(\cos\theta-1)$	**M1**	OE Or use of formula or completing the square. Allow use of replacement variable.
	$\cos\theta=\frac{2}{8}$, $\cos\theta=1$	**A1**	OE For both answers. **SC:** If M0, **SC B1** available for sight of $\cos\theta=\frac{2}{8}$ and 1
	[θ =] 0°, 75.5°	**A1**	AWRT ISW rejection of 0°. For both answers and no others in the range $0°\leqslant\theta\leqslant180°$, must be in degrees. **SC:** If M0 B1 scored, **SC B1** available for correct answers. **SC**: If M1 A0 scored, **SC B1** available for $\cos\theta=\frac{2}{8}$ and θ = 75.5° only, WWW.
		3	

Question	Answer	Marks	Guidance
4	$ar^2 = 1764$ and $ar + ar^2 = 3444$ or $ar = 1680$ or $\frac{a(1-r^3)}{1-r}-a=3444$	**B1**	Two correct algebraic statements.
	Attempt to solve as far as $r =$ or $a =$	**M1**	Any valid method, e.g. $1764\div1680$ or from $20r^2-41r+21$ OE (condone solving using a calculator).
	$r=\frac{1764}{1680}=\frac{21}{20}$ or 1.05 [a = 1600]	**A1**	Note: $r=\frac{1764}{3444-1764}$ www implies B1 and M1.
	17 500	**A1**	AWRT e.g. 17 474.1…..
		4	

Question	Answer	Marks	Guidance
5(a)	Three points at the bottom of their transformed graph plotted at $y = 2$	**B1**	All 5 points of the graph must be connected.
	Bottom three points of ⋀⋀ at $x = 0$, $x = 1$ & $x = 2$	**B1**	Must be this shape.
	All correct	**B1**	Condone extra cycles outside $0 \leqslant x \leqslant 2$.
		3	**SC:** If B0 B0 scored, B1 available for ⋀ in one of correct positions **or** all 5 points correctly plotted and not connected **or** correctly sized shape in the wrong position.
5(b)	[g(x) =] f(2x) + 1	**B1 B1**	Award marks for their final answer as follows: f(2x) B1, + 1 B1. Condone $y =$ or $f\left(x\right) =$.
		2	

Question	Answer	Marks	Guidance
6(a)	$\mathbf{y} = 4\left(x + \frac{5}{2}\right)^2 - 19$		There is no requirement for the candidate to list a , b and c. Look at values in their final expression, condone omission of ^{2} , and award marks as follows:
		B1	$a = 4$
		B1	$b = \frac{5}{2}$ OE
		B1	$c = -19$
		3	

Question	Answer	Marks	Guidance
6(b)	$\left(\textit{Their}\ 4\left(x+\frac{5}{2}\right)^2-19\right)=45\ \left[\Rightarrow\left(x+\frac{5}{2}\right)^2=16\right]$	*M1	Equate their quadratic completed square form from **6(a)** to 45 or re-start and use completing the square.
	Solve as far as $x=$	DM1	Any valid method leading to two answers.
	$[x=]\frac{3}{2},-\frac{13}{2}$	A1	**SC:** If M0 or M1 DM0 awarded, B1 available for correct final answers.
		3	
6(c)	Quadratic **curve** that is the right way up (must be seen either side of stationary point)	B1	No axes required, ignore any axes even if incorrect.
	Stationary point stated using any valid method or correctly labelled on their diagram.	B1 FT B1 FT	FT *their* values from **6(a)** as long as *their* expression is of the form $p(qx+r)^2+s$. Expect $\left(-\frac{5}{2},-19\right)$. Condone if stated correctly but plotted incorrectly.
		3	

Question	Answer	Marks	Guidance
7(a)	$\dfrac{\sin\theta(\sin\theta-\cos\theta)+\cos\theta(\sin\theta+\cos\theta)}{(\sin\theta+\cos\theta)(\sin\theta-\cos\theta)}\left[=\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta-\cos^2\theta}\right]$	*M1	Sight of a correct common denominator, either in one or two fractions, condone missing brackets if recovered. In the numerator condone $\pm$ sign errors only.
	$\dfrac{\dfrac{\sin^2\theta}{\cos^2\theta}+\dfrac{\cos^2\theta}{\cos^2\theta}}{\dfrac{\sin^2\theta}{\cos^2\theta}-\dfrac{\cos^2\theta}{\cos^2\theta}}$	DM1	Divide throughout by $\cos^2\theta$.
	$\dfrac{\tan^2\theta+1}{\tan^2\theta-1}$ AG	A1	
	Alternative method for Question 7(a)		
	$\dfrac{\dfrac{\sin^2\theta}{\cos^2\theta}+1}{\dfrac{\sin^2\theta}{\cos^2\theta}-1}\times\dfrac{\cos^2\theta}{\cos^2\theta}$ or the equivalent step $\left[=\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta-\cos^2\theta}\right]$	*M1	Replace $\tan^2\theta$ with $\dfrac{\sin^2\theta}{\cos^2\theta}$ and multiply top and bottom by $\cos^2\theta$. Condone $\pm$ sign errors.
	Sight of convincing use of partial fractions	DM1	
	$\dfrac{\boldsymbol{\sin\theta}}{\boldsymbol{\sin\theta+\cos\theta}}+\dfrac{\boldsymbol{\cos\theta}}{\boldsymbol{\sin\theta-\cos\theta}}$ AG	A1	
		3	**Note:** M1 DM1 A1 for working on both sides at the same time and finishing at the same correct expression. M1 DM1 for starting separately and finishing at the same correct expression and A1 if there is a final conclusion e.g. QED. Do not allow cross multiplication. Condone use of s, c and t and omission of θ.

Question	Answer	Marks	Guidance
7(b)	$\dfrac{\tan^2\theta+1}{\tan^2\theta-1}=2 \Rightarrow \tan^2\theta+1=2\left(\tan^2\theta-1\right)$	*M1	Equate expression from **(a)** to 2 and clear fraction.
	$\tan\theta=[\pm]\sqrt{3}$	DM1	Simplify as far as $\tan\theta=$. May be implied by a correct final answer in degrees or radians.
	Alternative method for first two marks of Question 7(b)		
	$\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta-\cos^2\theta}=2 \Rightarrow\ 1=2\sin^2\theta-2\left(1-\sin^2\theta\right)$	*M1	Equate expression to 2, clear fraction and use trig identities to form an equation in $sin\theta$ or $cos\theta$ only.
	$sin\theta=[\pm]\sqrt{\dfrac{3}{4}}$ or $cos\theta=[\pm]\sqrt{\dfrac{1}{4}}$	DM1	Simplify as far as $sin\theta=$, or $cos\theta=$.
	$\theta=\dfrac{1}{3}\boldsymbol{\pi},\dfrac{2}{3}\boldsymbol{\pi}$	A1 A1 FT	A1 for either correct answer then A1FT For their second value being $\pi-$ (their first) and no others in range $0\leqslant\theta\leqslant\pi$, both values must be exact and in radians. **SC:** B1 for $\theta=60°,120°$ or $0.333\pi,0.667\pi$ AWRT. or 1.05, 2.09 AWRT.
		4	

Question	Answer	Marks	Guidance
8(a)	$[y=]\left\{\dfrac{3x^{\frac{3}{2}}}{\frac{3}{2}}\right\}+\left\{-\dfrac{3x^{\frac{1}{2}}}{\frac{1}{2}}\right\}[+c]\left[=2x^{\frac{3}{2}}-6x^{\frac{1}{2}}\right]$	**B1 B1**	Marks can be awarded for correct unsimplified expressions, 1 mark each for contents of { } ISW.
	$5=2\times3^{\frac{3}{2}}-6\times3^{\frac{1}{2}}+c$	**M1**	Correct use of (3,5) in an integrated expression (defined by at least one correct power) including + c.
	$\boldsymbol{y}=2\boldsymbol{x}^{\frac{3}{2}}-6\boldsymbol{x}^{\frac{1}{2}}+5$	**A1**	Condone c = 5 as their final line if either y = or f(x) = seen elsewhere in the solution, but coefficients must not contain unresolved double fractions.
		4	
8(b)	$3x^{\frac{1}{2}}-3x^{-\frac{1}{2}}=0$	**M1**	Setting given differential to 0.
	[x=] 1	**A1**	CAO WWW Condone extra solution of —1 only if it is rejected.
		2	
8(c)	x>1 or x> "*their* 8(b)"	**B1FT**	Allow ⩾
		1	

Question	Answer	Marks	Guidance
9(a)	$\boldsymbol{a}\left(\boldsymbol{x}+\dfrac{1}{\boldsymbol{x}}\right)+1$	**B1**	ISW
		1	

Question	Answer	Marks	Guidance
9(b)	$a\left(2+\frac{1}{2}\right)+1=11$	**M1**	Substitute x = 2 into *their* expression from **(a)** and equate to 11. This may be done in 2 stages: f(2)= 2.5, $g(2.5)=11$.
	[a =] 4	**A1**	
		2	
9(c)	No,[because it is] not one-one	**B1**	Or other suitable explanation that may include one to many or many to one.
		1	
9(d)	$[\mathrm{g}^{-1}(x)]=\frac{x-1}{5}$ WWW	**B1**	Condone use of a instead of 5.
	$[\mathrm{g}^{-1}\mathrm{f}(x)=]\frac{x+\frac{1}{x}-1}{5}$ OE	**M1**	Correct combination of their $\mathrm{g}^{-1}(x)$ with given $\mathrm{f}(x)$ Condone use of a instead of 5.
	$\frac{\boldsymbol{x}^2-\boldsymbol{x}+1}{5\boldsymbol{x}}$ or $\frac{1}{5}\left(\boldsymbol{x}+\frac{1}{\boldsymbol{x}}-1\right)$ or $\frac{1}{5}\left(\boldsymbol{x}+\boldsymbol{x}^{-1}-1\right)$ OE ISW	**A1**	Must not contain unresolved fractions e.g. $\frac{x+x^{-1}-1}{5}$.
		3	
9(e)	The domain of f does not include the whole of the range of g. **Or** The range of g does not lie in the domain of f.	**B1**	Accept an answer that includes an example outside the domain of f, e.g. g(—1) = — 4 but for f, $x>0$.
		1	

Question	Answer	Marks	Guidance
10(a)	$2.5\times\frac{4\pi}{3}+2.24\times\frac{5\pi}{6}$ [= 10.47[2] + 5.86[4] or $\frac{10\pi}{3}+\frac{28\pi}{15}$]	B1	For either arc correct. Arc ARB could be AR+RB.
		M1	For adding two (or three) arc lengths using different radii and angles and nothing else. SOI
	16.34 or $\frac{26\pi}{5}$	A1	AWRT Condone 16.33 only.
		3	
10(b)	Area AOB = $\frac{1}{2}\times 2.5^2\sin\frac{2\pi}{3}$ [=2.706] Area APB = $\frac{1}{2}\times 2.24^2\sin\frac{5\pi}{6}$ [=1.254]	M1	For either ΔAOB or ΔAPB (AB = 4.33, h= 1.25, 0.58) or any other valid method.
	[Difference =] 1.45	A1	AWRT Condone 1.46 only.
		2	
10(c)	Area AOB = $\frac{1}{2}\times 2.5^2\times\frac{4\pi}{3}$ [=13.09] Area APB = $\frac{1}{2}\times 2.24^2\times\frac{5\pi}{6}$ [=6.57]	B1	For either sector area correct
	[Area of cross section =] $\frac{1}{2}\times 2.5^2\times\frac{4\pi}{3}+\frac{1}{2}\times 2.24^2\times\frac{5\pi}{6}+$ "$their\,10(b)$" $\left[=13.09+6.57+\text{"}their\,10(b)\text{"}\right]$	M1	Adding two **sector areas** from different sectors and '$their\,10$**(b)**' and nothing else. SOI
	21.1	A1	CAO Condone slight inaccuracies in intermediate working if the correct answer is arrived at.
		3	

Question	Answer	Marks	Guidance
11(a)	$\left[\frac{dy}{dx}=\right]\frac{9}{2}x-12$ [= 0] or [y =] $\frac{9}{4}\left\{\left(x-\frac{8}{3}\right)^2+\frac{8}{9}\right\}$ or $\frac{9}{4}\left(x-\frac{8}{3}\right)^2+2$	**B1**	OE Either $\frac{dy}{dx}$ or a correct expression in completed square form. Allow unsimplified.
	$x=\frac{24}{9}$	**B1**	OE Condone 2.67 AWRT.
	$y = 2$	**B1**	CAO Note: $x=\frac{-b}{2a}=\frac{8}{3}$ B1; substitute $\frac{8}{3}$ for x in y = B1; y =2 B1.
		3	

Question	Answer	Marks	Guidance
For 11(b) look for working to be marked on page 19 or annotate it as BP or SEEN			
11(b)	[Area =] $\int\left(18-\frac{3}{8}x^{\frac{5}{2}}-\left(\frac{9}{4}x^2-12x+18\right)\right)dx$	**M1**	Intention to integrate **and** subtract areas (either way around). Can be two separate functions or combined. Using y^2 scores 0/5 but condone inclusion of π except for the final mark.
	Note: Subtraction not required for these marks. Either separately $\left([18x]-\frac{3x^{\frac{7}{2}}}{8\times\frac{7}{2}}\right)$, $\left(\frac{9x^3}{4\times3}-\frac{12x^2}{2}[+18x]\right)$ Or combined $[18x]-\frac{3x^{\frac{7}{2}}}{8\times\frac{7}{2}}-\frac{9x^3}{4\times3}+\frac{12x^2}{2}[-18x]$	**B1,B1**	One mark for correct integration of each curve, allow unsimplified. $\left([18x]-\frac{3}{28}x^{\frac{7}{2}}\right)$ $\left(\frac{3}{4}x^3-6x^2[+18x]\right)$ or $[18x]-\frac{3}{28}x^{\frac{7}{2}}-\frac{3}{4}x^3+6x^2[-18x]$ **BUT** condone sign errors that are only due to missing brackets.
	$=\left(-\frac{3}{28}\times4^{\frac{7}{2}}-\frac{3}{4}\times4^3+6\times4^2\right)$ $[-(0)]$	**M1**	Clear substitution of 4 into at least one integrated expression (defined by at least one correct power) which can be unsimplified.
	$=\frac{240}{7}$ or **34.3** AWRT	**A1**	**SC:** If all marks awarded except the final M1, SCB1 is available for the correct final answer.
		5	

Question	Answer	Marks	Guidance
11(c)	$\left[\frac{dy}{dx}=\right]\frac{-5\times 3}{2\times 8}x^{\frac{3}{2}}\left[=-\frac{15}{16}x^{\frac{3}{2}}\right]$	**B1**	Allow unsimplified.
	$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}\Rightarrow\frac{dy}{dt}=-\frac{15}{16}\times 8\times 2$	**M1**	Substitute $x=4$ into their $\frac{dy}{dx}$ and multiply by 2.
	-15	**A1**	Accept decreasing [at/by] 15
		3	**Note:** If incorrect curve used, this is not a MR and only M1 mark is available. Expect $(\frac{9(4)}{2}-12)\times 2[=12]$